The durations of recession and prosperity: does their distribution follow a power or an exponential law?

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Following findings by Ormerod and Mounfield [1] Wright [2] rises the problem whether a power [1] or an exponential law [2] describes the distribution of occurrences of economic recession periods. In order to clarify the controversy a different set of GDP data is hereby examined. The conclusion about a power law distribution of recession periods seems better though the matter is not entirely settled. The case of prosperity duration is also studied and is found to follow a power law. Universal but also non universal features between recession and prosperity cases are emphasized. Considering that the economy is basically a bistable (recession/prosperity) system we may derive a characteristic (de)stabilisation time.

I. INTRODUCTION

Ormerod and Mounfield [1] have analysed data from 17 capitalist economies between 1870 and 1994 and concluded that the *number* of duration of recessions is consistent with a power law. Wright [2] claims that the data rather follows an exponential law. However the problem, i.e. which law is really governing the occurrence distribution of recessions was not solved by Wright [2] who has not proposed any additional explanations beyond a mere fit discussion. The controversy stems from the number of data points used in measuring the shortest time intervals for a recession. In order to justify which law(s) better describe(s) the frequency distribution of recession periods of a given duration a different set of data is hereby investigated. The present idea is to consider results using a "high frequency data set", i.e. considering quarterly, rather than annual periods as in [1, 2]. According to standard scaling range theories [3, 4] the Ormerod and Mounfield's hypothesis, if valid, should be observable also on different time scales.

In Sect. II the data source is described. The data analysis for recessions is found in Sect. III and that for prosperity durations in Sect. IV. Conclusions are found in Sect. V. Another type of two parameter fit is attempted, i.e. a double exponential in an Appendix.

II. DATA SOURCE

A recession, for a given country, has occurred when its GDP has decreased between ends of two consecutive time intervals. The recession duration may last several time intervals (a "period"). This definition is equivalent to that used by Ormerod and Mounfield [1]. Some economists might prefer that periods with actual growth above a time-averaged growth rate G per year or per quarter should be counted as prosperity, and those with growth below G as recession. This surrogate data might lead to other conclusions, but no such investigation has been made at this time.

The complementary set of data points is the so called set of *prosperity* occurrences. In both cases no specification is hereby made on the "strength" of the recession or prosperity. In other words the time series is considered to be a set based on two characters (or + and - signs) similar in physics to a magnetic (up or down) spin chain or in informatics to a series of (0, 1) bits. In physics words, we consider the total number of spins in the chain, the number of spin domains and their size (the number of domain walls is the number of recession occurrences). The analysis of their distribution has been performed in the same spirit as in [1]. However for the sake of comparison, the different data sets are normalised such that it is the frequency f of a recession occurrence time interval which is examined rather than the number of occurrences.

Ormerod and Mounfield [1] and Wright [2] have analysed the GDP *annual* records of 17 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, New Zealand, Sweden, Switzerland, United Kingdom and USA, i.e. a total of 1965 data points, over 124 years, i.e. between 1870

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and 1994. Ormerod and Mounfield [1] found a power law dependence for the 336 occurrence of recessions, lasting *in toto* 541 years (y) (Tables 1 and 2). Wright [2] reached a quite different conclusion.

In order to verify the findings it is useful to examine a different set of data within different time limits. Therefore GDP data for *quarter* period reports are hereby examined. The data is taken from the Organisation for Economic Co-operation and Development (OECD) web page [5], where quarterly GDP data are found over 14 years from 1989 Q1 to 2003 Q2 (N.B. Q1, Q2, Q3, Q4 denotes first, second, third and fourth quarter of a year respectively.) for 21 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Great Britain and USA, amounting to a total of 1100 data points, distributed over 213 quarters (Q) for 136 recession occurrences (Tables 1 and 2). (N.B. In the case of Germany the data are only available from 1991 Q1, Portugal from 1995 Q1 and Sweden from 1993 Q1.)

Within the paper the following notation will be used: "y" for year and "Q" for quarter; GDP17 is the data studied in [1, 2], the so called low frequency data, while GDP21 is the so called high frequency data, used for the present study.

The statistical properties of the low frequency (y^{-1}) and high frequency (Q^{-1}) recession duration distributions are given in Table I. At this time there are more data points in the former set, since the Q recording is more recent. However the statistical properties look quite similar. The recession duration distribution mean, variance, kurtosis, skewness and entropy have the same order of magnitude. Recall (Table I) that the statistical entropy is defined as:

$$S = -\sum f_i \ln(f_i). \tag{1}$$

III. RECESSION ANALYSIS

Notice first that a statistical analysis and tests performed on shuffled data, i.e. in the case of an equivalent in size set of purely stochastically independent data taken from a skewed binomial distribution taking into account the empirically found relative probability of recession or prosperity, implies that such a distribution is described by an exponential law. This can be understood since in the case of independent events the probability of registering the sequence of length n of identical events is $p(n) = P^n$, where P is the probability of the occurring event.

A. Low frequency GDP17 data

The data collected by Ormerod and Mounfield [1] as recalled in Table II is presented on a log-log and a semi-log plot in Fig.1(a) and Fig.1(b) respectively with the best fitting straight line in both cases, corresponding to

$$f(d) = \gamma d^{-\delta} \tag{2}$$

and

$$f(d) = \alpha e^{-\beta d}. \tag{3}$$

The power law in Eq.(2) can be alternatively written as

$$f(d) = (\hat{\gamma} d)^{-\delta}. \tag{4}$$

The correlation coefficients have been calculated for both the semi-log and log-log transformations; see Table III for their value and those of the theoretical formula parameters.

In the case of data presented in [1, 2] the best correlation coefficient (Table III) occurs for $R_{GDP17,7_{semi-log}} = -0.993$ to be contrasted to the value $R_{GDP17,7_{log-log}} = -0.976$, supporting the hypothesis [2] that the data follows an exponential law.

If two (here, rather than one as in [2]) data points corresponding to the longest and the shortest periods are dropped, a linear fit on the log-log plot appears significantly better (see dash line in Fig.1(b)). Indeed the correlation coefficients take values $R_{GDP17,5_{semilog}} = -0.983$, and $R_{GDP17,5_{loglog}} = -0.9996$ respectively, suggesting that the shortest data follows a power law, as in [1].

Thus the case of short and long durations should receive better attention. Due to the scarcity of data points only a shorter time scale is convenient. Indeed very "long period" cases, like over decades, are not easily available nor numerous. Moreover it might be necessary to distinguish them according to the depth of the recession. In fact the (annual) data contains only one such a case (Table II). Thus we consider only "higher frequency data" even though such (quarterly) GDP reports are necessarily less numerous, since in fact they belong to the most recent times.

	GDP 17 recession (time=y)	GDP 21 recession (time=Q)	GDP 21 prosperity (time=Q)
Data length (time)	541	213	887
Number of occurrences	336	136	144
Mean (time)	1.610	1.566	6.160
Variance ($time^2$)	1.003	1.092	50.876
Kurtosis ($time^3$)	2.268	2.346	2.766
Skewness ($time^4$)	9.114	9.328	12.131
Entropy	1.06	1.0	2.57

TABLE I: Statistical properties of low [1] and high frequency [5] GDP data

GDP17, low frequency data (1965 data points)

Duration (y) of recessions	1	2	3	4	5	6	7
Number of recessions	206	88	23	10	5	3	1
Frequency	0.613	0.262	0.068	0.030	0.015	0.009	0.003

GDP21, high frequency data (1100 data points)

Duration (Q) of recessions	1	2	3	4	5	6	7
Number of recessions	93	23	12	4	3	0	1
Frequency	0.684	0.169	0.088	0.029	0.022	0	0.007

TABLE II: Durations of recessions. The low [1] and high frequency GDP data [5]

B. High frequency GDP21 data

The GDP data for the occurrences of recessions and their duration is presented in Table II for 21 countries [5]. As in the case of data used in [1] the highest number (and frequency) of recessions is registered for the case of the shortest period. The longest recession (7 Q) occurred in Finland; there are several five quarter long recessions exactly corresponding to those in the yearly data (Fig. 2). The statistical characteristics of the recession period data are presented in Table I. The overall relationship between the duration and occurrence of recessions seems to be similar in both (high and low frequency) sets of data, though the quarterly data appears to be more peaked (Table II).

The collected data are presented in semi-log (Fig.3(a)) and log-log (Fig.3(b)) plots together with the best fitting lines. The parameters of Eqs. (2)- (3) and the correlation coefficients are given in Table III and Table IV both for the semi-log and log-log fits. Comparing the values of the correlation coefficients $R_{GDP21\_semilog} = -0.823$ and $R_{GDP21\_loglog=-0.937}$ (Table III) we may conclude that the data from [5] better follows a power law as found in [1].

C. Truncated data

Following the ideas of [2], let us remove the rarest case. The fits (Fig. 3) are not changing significantly; the correlation coefficients (Table III) do not much improve : 0.82 $\rightarrow$ 0.86, 0.94 $\rightarrow$ 0.92. However, since $|R_{GDP21,5\_log-log}| < |R_{GDP21,6\_log-log}|$, (0.918 $<$ 0.937) in this case, a strong argument for the power law seems to hold for the GDP21 data. The parameters of the laws (Eq. (2) - (3)) are given in Table III as well.

IV. PROSPERITY ANALYSIS

Investigating the problem of dependencies between the frequency of occurrence and the duration of recessions another question arises: is there any law governing prosperity duration times?

A period is treated as a prosperity one if the GDP has increased between the end of two consecutive time intervals. The prosperity periods are complementary to the recession periods. There are 887 positive Q's (or "up-spins") distributed into 144 periods ("domains"), Table I. E.g. the prosperity duration may be much longer than recession durations, as up to 44 Q (for Great Britain). The occurrence of durations of prosperity periods is presented as a

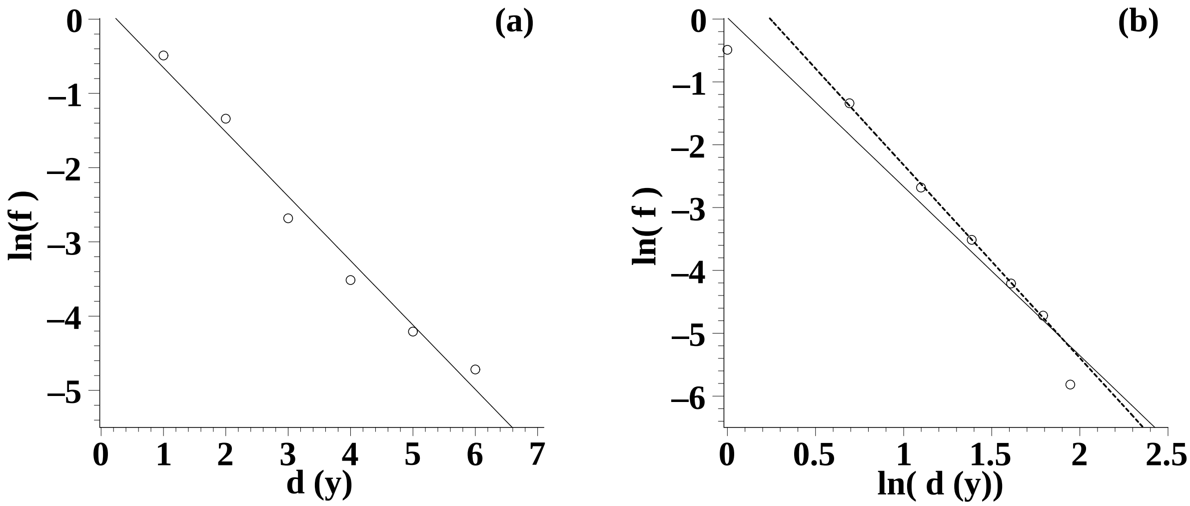

FIG. 1: Data examined in [1, 2] in (a) semi-log, (b) loglog plot. Solid line, the best fit to all data; dashed line, the best fit to the data without the longest and the shortest durations included, as in [2]

	GDP 17 recession $[time = y]$		GDP 21 recession $[time = Q]$	
data points	7	5	6	5
R (*semi-log*)	-0.993	-0.983	-0.823	-0.858
R (*log-log*)	-0.976	-0.9996	-0.937	-0.918
$\ln(\alpha)$	0.21 ± 0.21	0.02 ± 0.38	-0.13 ± 0.33	0.20 ± 0.34
α	1.24	1.02	0.88	1.22
$\beta[time^{-1}]$	0.87 ± 0.05	0.83 ± 0.09	0.73 ± 0.08	0.86 ± 0.10
δ	2.69 ± 0.27	3.07 ± 0.05	2.30 ± 0.14	2.17 ± 0.18
$\ln(\gamma)$	0.02 ± 0.37	0.75 ± 0.07	-0.22 ± 0.19	-0.31 ± 0.18
$\gamma[time^{-\delta}]$	1.02	2.11	0.80	0.73
$\hat{\gamma}[time^{-1}]$	0.99	0.78	1.10	1.16
$1/\zeta(\delta)$	0.78	0.84	0.69	0.66

TABLE III: Recession data; fitting parameters

histogram in Fig. 2 while the statistical properties are given in Table I. The mean value of the prosperity periods is about four times longer than in the case of the recession durations, for the time intervals examined here.

The data is presented in semi-log (Fig.4(b)) and log-log (Fig.4(a)) plots with the best fitting lines. As in the case of the recession time distribution, the correlation coefficients have been calculated together with the values of the parameters appearing in the functions Eq.(3) and Eq.(2). The results are found in Table IV.

The investigated data contains eight cases (durations (Q) = 16; 17; 24; 25; 28; 31; 40; 44) where only one occurrence of such a size is registered. The fit has been repeated on the corresponding truncated data set, in line with [2] idea. The results are found in Table IV. The fit precision does not (to say the least) increase, and a power law still better describes the data. The parameters and regression coefficients of the power law Eq.(2) are given in Table IV) as well.

	GDP 21 prosperity	
data points	23	15
R (*semi-log*)	-0.976	-0.980
R (*log-log*)	-0.992	-0.992
$\ln(\alpha)$	-2.55 ± 0.23	-1.97 ± 0.23
α	0.08	0.14
$\beta[Q^{-1}]$	0.079 ± 0.012	0.13 ± 0.022
δ	1.12 ± 0.09	1.01 ± 0.12
$\ln(\gamma)$	-1.12 ± 0.23	-1.23 ± 0.25
$\gamma[Q^{-\delta}]$	0.33	0.29
$\hat{\gamma}[Q^{-1}]$	2.43	2.74
$1/\zeta(\delta)$	0.11	0.01

TABLE IV: Prosperity data; fitting parameters

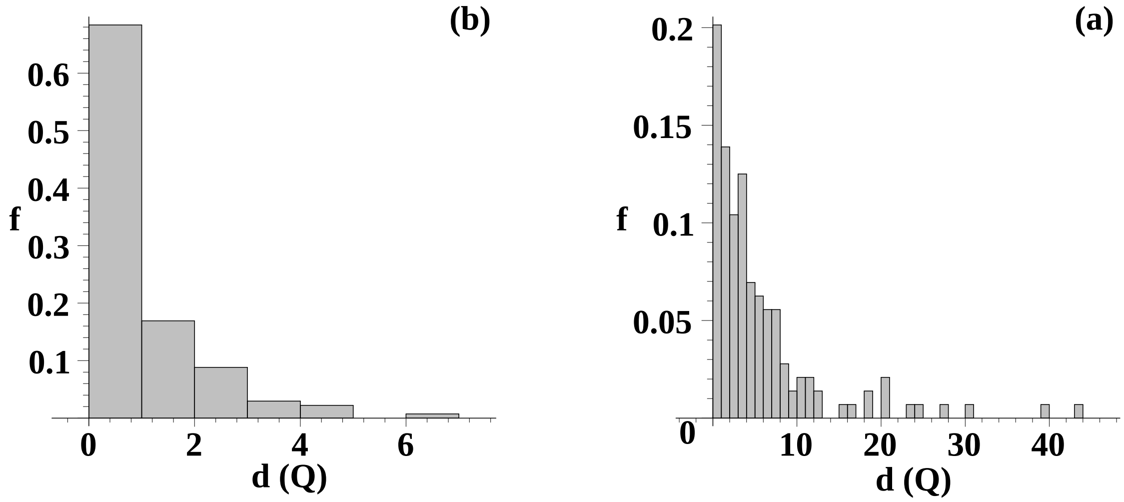

FIG. 2: (a) Histogram of recession durations observed in the GDP21 data [5]; (b) Histogram of the prosperity periods

V. CONCLUSIONS

The observed correlations between the duration of a recession (or prosperity) periods and their probability of occurrence is hereby interesting. It appears that the relationship is nontrivial. First the values of the exponents β^{-1} and $\hat{\gamma}^{-1}$ give an information on the "relaxation time" of the processes. It is roughly 1.0 (y or Q) for *recessions*; this is a remarkable scale free, whence so called universal, result. However the corresponding "relaxation time" is an order of magnitude different, i.e. 10 Q for *prosperity* cases. There is a major difference concerning the *decay exponent* δ though: δ is about -2.5 for recessions, but $\sim$ -1.1 for the prosperity cases. Also, $\hat{\gamma}^{-1}$ is quite different from β^{-1} for the prosperity cases. This points again to non universal features (or to different universality classes ?).

From an economic point of view the above relaxation time values should not be confused with the apparent periodicity (or better "periodicities") of business cycles [6–12]. The latter ones are rather to be compared to the means found in Table 1. Even these seem small with respect to so called "common feeling", which rather measures a median more than a mean. In some sense this indicates the difference between a psychological or even visual data filtering based on the slowliest trend, mathematically a large window-moving average, in contrast to the actual works looking at somewhat higher frequency data.

Along this psychological line of expectations for economy periods, Hohnisch et al. [13] discuss the ups and downs

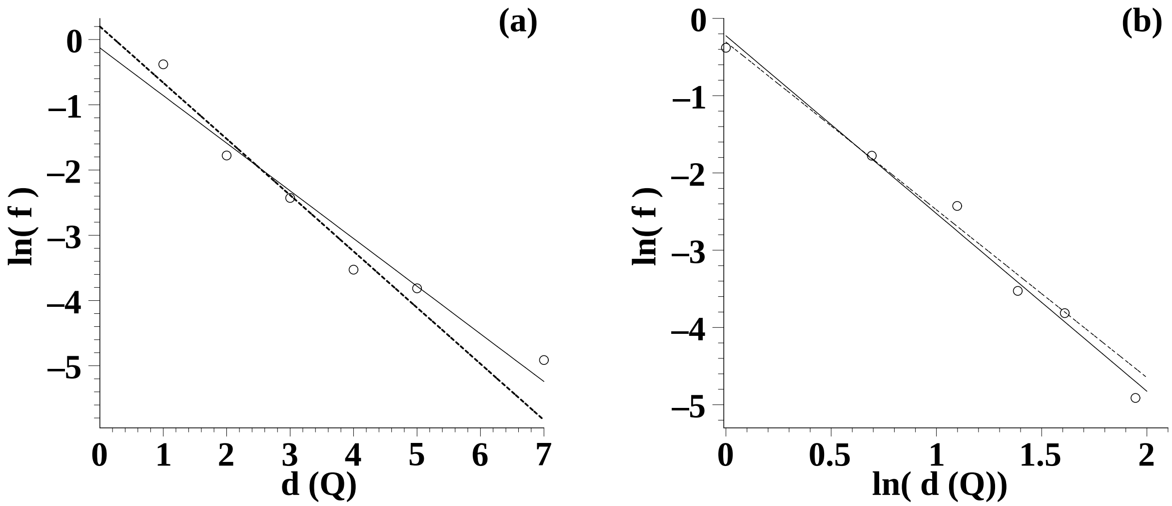

FIG. 3: Recession. Collected GDP21 data in (a) a semi-log and (b) log-log plot together with the fitted line. Solid line - all data, dashed one - the data without taking into account the longest recession duration

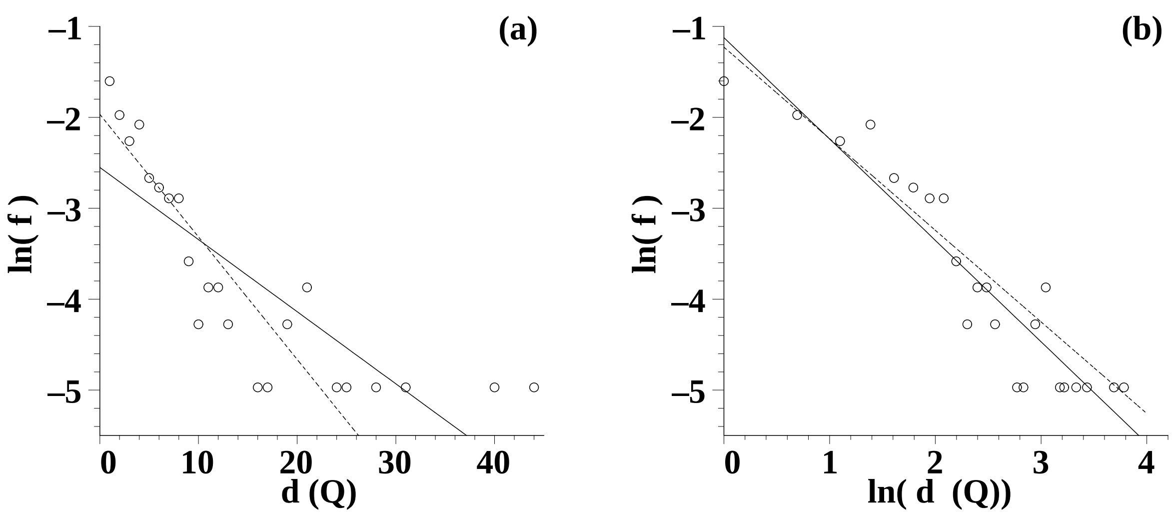

FIG. 4: Collected GDP21 data of prosperity duration occurrences in (a) semi-log and (b) log-log plot together with the fitted line. Solid - all data, dashed one - the data without the cases where only one events was registered

of *opinions* about recessions and prosperity along a Blume-Capel model and Glauber dynamics. Their computer simulations show that opinions drastically undergo changes from one equilibrium to the other, both having rough but small fluctuations, - as in stochastically resonant systems.

Nevertheless, except for the debatable GDP17 full data set, considering the results obtained in the above sections III and IV it is observed that a power law significantly better describes the distribution of durations of both recession and prosperity periods. For recessions, another type of two parameter fit can be also attempted, i.e. a double exponential, see Appendix.

A final argument in favor of a true power law distribution should follow from the measure of the γ parameter which should be equal to $1/\zeta(\delta)$ where $\zeta(\delta)$ is the Riemann ζ function. The $1/\zeta(\delta)$ values are given as the last line in each

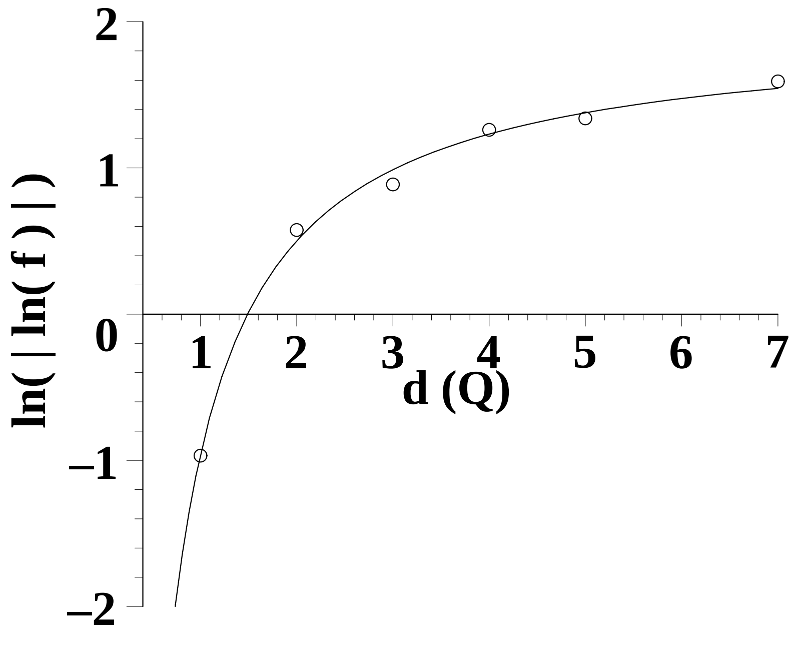

FIG. 5: Collected GDP21 data of recession duration frequency occurrences in normal-loglog plot together with the fitted curve, $|ln(f)| = \lambda e^{-\frac{\mu}{d}}$

Table. It is remarkable in view of the available data and the error bars that there is some reasonable agreement for the recession cases, the more so for the high frequency data. The prosperity cases do not seem to obey such a criterion. This is of course due to the huge tail of rare (though very long) prosperity events during the last years.

In summary the hypothesis stated by Ormerod and Mounfield seems to be fine for recession occurrences though a theory is still needed, but the case of prosperity durations is not settled, due to the very long and low tail of the distribution.

The power law hereby preferred thus shows some coupling between successive (economy or opinion or..) swings is missing in the (simple) computer model of Hohnisch et al. [13], like the (simple) random walk model of Bachelier [14] which gives a Gaussian or log-normal distribution of the price fluctuations, needs a non-random explanation, like psychological herding etc. [15] for getting as actually here the observed power law distribution. It is finally fair to mention that such successions of recessions and prosperity periods, sometimes called business cycles, have recently received a renewed deal of attention from analytical and simulation points of view [10–12, 16–20] beside theoretical work in classical macroeconommic studies [21].

Appendix

Another type of two parameter fit can be also attempted, i.e. a double exponential (ee):

$$|ln(f)| = \lambda e^{-\frac{\mu}{d}}, \tag{5}$$

where f denotes the occurrence frequency and d the duration of a (recession or) prosperity period, while μ, and λ are unknown coefficients. In the case of GDP21 recession data: $\mu = 2.92$ Q, and $\lambda = 7.11$ while the correlation coefficient is $R_{ee\_loglog} = -0.998$, - the best of all attempted fits when the full data is used in the main text. The result seems to be interesting, but there is no theoretical interpretation of it. However notice that the formula looks like that corresponding to an Arrhenius formula in chemistry where the μ coefficient corresponds to an activation energy, when d is the temperature.

This (Arrhenius-like) formula usually applies when some system balances between a stable state and an excited state. In that spirit one might consider that the value of $\mu \simeq 3Q$ is of the same order of magnitude as the value (1 Q) found in the main text for the power law, and is in the range estimated as a realistic time necessary for responding to some (*de*)*stabilisation* constraint of the economic field. The double exponential form in financial data has received some consideration in [22].

VI. ACKNOWLEDGEMENTS

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